Corrigendum to “Reconciling the Rawlsian and the utilitarian approaches to the maximization of social welfare” [Economics Letters 122 (2014) 439–444]

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Subsection 3.1. (pp. 441–442) of the originally published article analyzes the optimal choice of a Rawlsian social planner (RSP). The originally published subsection did not cover all possible cases, and the proof that a RSP will choose to equalize incomes is incomplete. The text that follows replaces that subsection.

3.1. The maximization problem of a Rawlsian social planner

The maximization problem of a RSP is

$$\max_{\Omega(a_1,\ldots,a_n;\lambda)} \text{SWF}_R(x_1,\ldots,x_n)$$

$$= \max_{\Omega(a_1,\ldots,a_n;\lambda)} \{\min \{u_1(x_1,\ldots,x_n),\ldots,u_n(x_1,\ldots,x_n)\}\}.$$ (3)

It is easy to see that for every $$k \in \{1,\ldots,n-1\}$$ we have that

$$u_i(x_1,\ldots,x_k,x_{k+1},\ldots,x_n) = u_i(x_1,\ldots,x_{k+1},x_k,\ldots,x_n)$$

for $$i \in \{1,\ldots,n\} \setminus \{k,k+1\}$$, and that

$$u_k(x_1,\ldots,x_k,x_{k+1},\ldots,x_n) = u_{k+1}(x_1,\ldots,x_{k+1},x_k,\ldots,x_n).$$

Therefore, if $$x_1 \leq \cdots \leq x_n$$, then the monotonicity of the $$f$$ function and the definition of the $$RI$$ function imply that $$u_1(x_1,\ldots,x_n) \leq u_2(x_1,\ldots,x_n) \leq \cdots \leq u_n(x_1,\ldots,x_n).$$ Thus, for any $$k$$ such that $$y_k = \min\{y_1,\ldots,y_n\}$$, we have that $$\text{SWF}_R(y_1,\ldots,y_n) = u_k(y_1,\ldots,y_n).$$

Denoting by $$(x_1^{x^{*}},\ldots,x_n^{x^{*}})$$ the optimal post-transfer distribution of incomes of a RSP, we have that

$$\max_{\Omega(a_1,\ldots,a_n;\lambda)} \text{SWF}_R(x_1,\ldots,x_n) = u_1(x_1^{x^{*}},\ldots,x_n^{x^{*}}),$$

where $$x_1^{x^{*}} = \cdots = x_n^{x^{*}}$$. We prove this claim by contradiction. To do that, we assume that $$(x_1^{x^{*}},\ldots,x_n^{x^{*}}) \in \Omega(a_1,\ldots,a_n;\lambda)$$ is such that $$x = \min\{x_1^{x^{*}},\ldots,x_n^{x^{*}}\} < \max\{x_1^{x^{*}},\ldots,x_n^{x^{*}}\}$$, and that there exists $$(y_1,\ldots,y_n) \in \Omega(a_1,\ldots,a_n;\lambda)$$ such that $$\text{SWF}_R(y_1,\ldots,y_n) > \text{SWF}_R(x_1^{x^{*}},\ldots,x_n^{x^{*}})$$. Therefore, $$(x_1^{x^{*}},\ldots,x_n^{x^{*}})$$ cannot be a maximum.

Let $$I = \{i \in \{1,\ldots,n\} : x_i^{x^{*}} = x \land x_i^{x^{*}} \geq a_i\}, J = \{i \in \{1,\ldots,n\} : x_i^{x^{*}} = x \land x_i^{x^{*}} < a_i\}, h = \min\{i \in \{1,\ldots,n\} : x_i^{x^{*}} = x\}, K = I \cup J$$, and the notation $$|A|$$ stands for the cardinality of the set $$A$$. Obviously, from the characteristics of the point $$(x_1^{x^{*}},\ldots,x_n^{x^{*}})$$, it follows that $$I \neq \emptyset$$ and that $$h \geq 1$$.

Let $$\delta$$ be such that $$0 < \delta < \min\{\frac{\lambda(x-h)}{2}, \min_{i \in K, a_i < x_i^{x^{*}}} \{a_i - x_i^{x^{*}}\}\}$$. We now define the coordinates of the point $$(y_1,\ldots,y_n)$$ as

$$y_i = \begin{cases} x_i^{x^{*}} + \frac{\delta}{h} & \text{for } i \in I \cup J, \\ x_i^{x^{*}} - \delta_k & \text{for } i = k, \\ x_i^{x^{*}} & \text{for } i \in \{1,\ldots,n\} \setminus K, \end{cases}$$

where $$\delta_k = \delta(|I| + |J|)/(|I| + \lambda |J|)$$ if $$x_k^{x^{*}} \leq a_k$$ and $$\delta_k = \delta(|I| + \lambda |J|)/h$$ otherwise. It is easy to verify that $$(y_1,\ldots,y_n) \in \Omega(a_1,\ldots,a_n;\lambda)$$.

Because the $$f$$ function is an increasing function, and because a smaller difference between incomes implies a smaller value of the index of low relative income, it follows that for any $$i \in I \cup J$$

$$\text{SWF}_R(y_1,\ldots,y_n) - \text{SWF}_R(x_1^{x^{*}},\ldots,x_n^{x^{*}}) = u_i(y_1,\ldots,y_n) - u_i(x_1^{x^{*}},\ldots,x_n^{x^{*}})$$

$$= (1 - \beta) \left[ f(x_i^{x^{*}} + \delta/h) - f(x_i^{x^{*}}) \right] - \beta \left[ RI(x_i^{x^{*}} + \delta/h; y_1,\ldots,y_n) - RI(x_i^{x^{*}}; x_1^{x^{*}},\ldots,x_n^{x^{*}}) \right] > 0$$

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for any $\beta \in [0,1)$ and $0 < \lambda \leq 1$. Therefore, $SWF_\beta(y_1, \ldots, y_n) > SWF_\beta(x_1^{R^*}, \ldots, x_n^{R^*})$, which contradicts the fact that $SWF_\beta$ attains a global maximum at $(x_1^{R^*}, \ldots, x_n^{R^*})$. Thus, the solution of the problem of a Rawlsian social planner, (3), has to be a transfer such that the post-transfer incomes are all equal. This completes the proof by contradiction.

It is worth noting that the solution of (3) is unique. To show this, we assume that $a_1 < a_n$, and we let

$$g(x) = \sum_{i=1}^{n} \frac{\max(x-a_i, 0)}{\sum_{i=1}^{n} \max(a_i-x, 0)}$$

for $x \in [a_1, a_n)$. Then, as a ratio of a continuous, strictly increasing function and a continuous, strictly decreasing and positive function, $g$ is continuous and strictly increasing, and $g(a_1) = 0$, $\lim_{x \to a_n} g(x) = \infty$. Therefore, there exists a unique $x^{R^*} \in (a_1, a_n)$ such that $g(x^{R^*}) = \lambda$, which is the solution of $\lambda \sum_{i=1}^{n} \max(a_i-x, 0) = \sum_{i=1}^{n} \max(x-a_i, 0)$, and we have that $x^{R^*} = x_1^{R^*} = \ldots = x_n^{R^*}$.

Concluding this subsection, we note that the distribution chosen by a RSP entails equality of incomes even when $\beta = 0$, namely, even if individuals’ concern at having low relative income is excluded from the RSP’s social welfare function.