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Turning relative deprivation into a performance incentive device

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ABSTRACT

The inclination of individuals to improve their performance when it lags behind that of others with whom they naturally compare themselves can be harnessed to optimize the individuals' effort in work and study. In a given set of individuals, we characterize each individual by his relative deprivation, which measures by how much the individual trails behind other individuals in the set doing better than him. We seek to divide the set into an exogenously predetermined number of groups (subsets) in order to maximize aggregate relative deprivation, so as to ensure that the incentive for the individuals to work or study harder because of unfavorable comparison with others is at its strongest. We find that the solution to this problem depends only on the individuals' ordinally measured levels of performance independent of the performance of comparators.

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> "Relative success, tested by an invidious pecuniary comparison with other men, becomes the conventional end of action. The currently accepted legitimate end of effort becomes the achievement of a [less un]favourable comparison with other men." (Veblen, 1899.)

1. Introduction

We build on the finding that people are motivated to perform better when their achievements lag behind the achievements of comparators (those with whom people naturally compare themselves). Drawing on a constructive example, we show how this tendency can be used to design a combination of allocations of individuals that maximizes the individuals' aggregate incentive to improve performance.

Our constructive example is based on three considerations: there is strong evidence of "a comparators' performance effect" (the performance of an individual, in particular effort exerted in study, work, and so on, is affected by the performance of comparators); the effect is asymmetrical (it is affected by those to the right of the individual in the relevant distribution); and we define a concrete cardinal measure that enables us to quantify the intensity of the effect (an index of relative deprivation).

With regard to the first two considerations, there is ample empirical evidence that the presence of better-performing comparators motivates students to perform better (examples are studies by Sacerdote, 2001; Azmat & Iriberri, 2010; Garlick, 2018), and workers to exert more effort (examples are studies by Falk & Ichino, 2006; Mas & Moretti, 2009; Bandiera, Barankay, & Rasul, 2010; Cohn, Fehr, Herrmann, & Schneider, 2014). With regard to the third consideration: in Appendix B we present a concise historical account of how the sociological-psychological concept of relative deprivation is linked to the discipline of economics, and we describe how we construct the index of relative deprivation that we use in this paper. In a nutshell, the relative deprivation experienced

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by a member of a population is the aggregate of the excess of the levels of performance of the member's comparators, divided by the size of the population.

In our own research, we have combined these three considerations. Here we list merely a few examples. In one study (Stark, 1990) we showed how the incentive to improve performance can be harnessed to design reward structures in career games and other contexts such as golf tournaments. In another study (Stark & Hyll, 2011) we analyzed the impact on a firm's profits and optimal wage rates, and on the distribution of workers' earnings, when workers compare their earnings with those of coworkers. We considered a low-productivity worker who receives lower earnings than a high-productivity worker. We showed that when the low-productivity worker derives (dis)utility not only from his own effort but also from comparing his earnings with those of the high-productivity worker, his response to the sensing of relative deprivation is to increase his optimal level of effort. Consequently, the firm's profits are higher, its wage rates remain unchanged, and the distribution of earnings is compressed. And in yet another study (Stark & Budzinski, 2019) we inquired how in the wake of migration from a community, say a village economy, the changes of the income distribution and the social comparison space in the village set in motion behavioral responses of the non-migrants, including changes in their work effort and, as a consequence, their output, and we showed whose migration will bring about the strongest incentive of the non-migrants to increase their work effort and output.

In this paper, rather than document the strength of the effect of the performance of others, we assume the effect, and we ask how acknowledging the effect can be exploited as a management tool, namely as a means of setting optimal incentives to improve performance.

We study a setting in which individuals who differ in their capability but are homogeneous in preferences (as shown below, they all exhibit the same distaste for relative deprivation) need to be distributed between an exogenously predetermined number of facilities, where the number of positions in each of the facilities differs by no more than one.

A few examples of this assignment task are presented next.

Suppose that we have two classes and four students. An exogenously imposed constraint is that the classes should be of equal size. The justifications for that are so as to equalize the study environments, and to ensure that no class can accommodate more than two students. We have two teachers on the school payroll, and all the students need to be schooled. How do we distribute students 4, 3, 2, and 1 between the two classes so that the incentives to study harder will be maximized? The numbers 4, 3, 2, and 1 represent levels of performance that are independent of the performances of comparators, namely how each student performs in isolation from the pressure of the comparators' performances.

In a supermarket, there are two exits at the two ends of the shop, each with two cash desks, and there are four cashiers on the payroll. The earnings of a cashier are determined, in part, by the number of grocery items processed. Cashiers observe each other at the same exit, but not across both exits.

There are three fields, at a distance one from the other, and in each field three harvesters are stationed. There are nine qualified harvester operators. The payment to a harvester operator is determined, in part, by the weight of the harvested crop. The fields differ, so performance comparisons are field-specific.

There is a production line in each of four car production plants. The engineers who designed the lines made them identical in terms of the positions to be manned. There are as many qualified assembly workers as there are positions. Because the cars produced differ markedly between the production sites, performance comparisons are site-specific.

The postal service serves six neighborhoods in Cambridge, MA. In terms of the characteristics and the type of services that the residents demand, the neighborhoods differ. For reasons of security, a post office branch cannot be manned by just one person, and closing a branch is not allowed. Part of the payment to a postal employee is performance-related, and given the distinct character of the six neighborhoods, comparisons of performance across branches are not relevant.

Finally, consider further the assignment problem in the case of two school classes with students 4, 3, 2, and 1. The assignment options are {{4,3},{2,1}}; {{4,2},{3,1}}; and {{4,1},{3,2}}. We assume that the ordering of the classes is immaterial, namely we treat {{4,3},{2,1}} and {{2,1},{4,3}} as the same option. Intuition suggests that the first of these three assignment options is dominated by the second and third assignment options: the aggregate "pressure" to improve performance appears to be higher in the case of the second and third assignments than in the case of the first assignment. In other words: if we assume that the incentive to study harder increases with the difference between the students in their free-from-comparison levels of performance,[1] then the division {{4,3},{2,1}} does not maximize the incentive to study harder because for the divisions {{4,2},{3,1}} and {{4,1},{3,2}}, the difference is twice as large.

2. A model of assignments aimed at maximizing the incentive to perform better

In order to formalize and generalize what the above examples entail, and in particular what the preceding school classes assignment tells us, we introduce some notation and three definitions.

Let $N = \{1, 2, \ldots, n\}$ be a set of individuals, $n \geq 4$, and $a_i \in \mathbb{R}_+$ is the comparison-free performance of individual $i \in N$ (such as the individual's initial test score). Without loss of generality, we assume that $a_1 < a_2 < \ldots < a_n$. Let $k \in \mathbb{N}$ be such that $k < n$. Denote $q = \lfloor \frac{n}{k} \rfloor$, and let $r \in \mathbb{N}$ be such that $n = kq + r$. These notations mean that when we divide N into k groups of equal size or of equal size but for one, we obtain r groups of $q + 1$ individuals, and $k - r$ groups of q individuals.

Definition 1.
A division of the set N into k groups is a family of sets $\{X_1, X_2, \ldots, X_r, Y_1, Y_2, \ldots, Y_{k-r}\}$ satisfying the following three conditions.

(i) $X_1, X_2, \ldots, X_r, Y_1, Y_2, \ldots, Y_{k-r}$ are pairwise disjoint;

(ii) $\bigcup_{i=1}^{r} X_i \cup \bigcup_{i=1}^{k-r} Y_i = N$;

(iii) $|X_i| = q + 1$ for $i \in \{1, 2, \ldots, r\}$[2] and $|Y_i| = q$ for $i \in \{1, 2, \ldots, k - r\}$, where the notation $|Z|$ stands for the number of elements in the set Z.

We use the notation of an unordered sequence $\{X_1, X_2, \ldots, X_r, Y_1, Y_2, \ldots, Y_{k-r}\}$ rather than the notation of an ordered sequence $(X_1, X_2, \ldots, X_r, Y_1, Y_2, \ldots, Y_{k-r})$ because in our context (for example, as already noted in the assignment problem of school classes) the ordering of the sets is immaterial. Conditions (i) and (ii) of Definition 1 state that each individual from N is assigned to exactly one element of the division of N (namely to one of the sets that belong to the family $\{X_1, X_2, \ldots, X_r, Y_1, Y_2, \ldots, Y_{k-r}\}$). Condition (iii) of Definition 1 requires that the individuals are distributed between the sets $X_1, X_2, \ldots, X_r, Y_1, Y_2, \ldots, Y_{k-r}$ equally, or equally but for one if $\frac{n}{k}$ is not a natural number. In light of typical real-life considerations, condition (iii) is reasonable. For example, students are usually divided into classes of (approximately) the same size so as to create similar learning environments.

Definition 2.
Let $\{X_1, X_2, \ldots, X_r, Y_1, Y_2, \ldots, Y_{k-r}\}$ be a division of N. The relative deprivation, RD, of individual $i \in S$ where $S \in \{X_1, X_2, \ldots, X_r, Y_1, Y_2, \ldots, Y_{k-r}\}$ is defined as

$$RD_S(i) \equiv \frac{1}{|S|} \sum_{j \in S} \max\{a_j - a_i, 0\}.$$

A detailed derivation of $RD_S(i)$ is in Appendix B.

[1]This argument is broadly in line with Akerlof (1997).
[2]Here and henceforth, if $r = 0$, then $\{1, 2, \ldots, r\} = \emptyset$.

Using aggregate relative deprivation (*ARD*) as a measure of the combined "pressure" to improve performance, we seek to maximize the function

$$ARD(\{X_1, X_2, \dots, X_r, Y_1, Y_2, \dots, Y_{k-r}\}) \equiv \sum_{S \in \{X_1, X_2, \dots, X_r, Y_1, Y_2, \dots, Y_{k-r}\}} \sum_{i \in S} RD_S(i).$$

Definition 3.
For a given set N and an ordered vector of performances that are independent of the performance of comparators $(a_1, a_2, \dots, a_n)$, a division $\{X_1, X_2, \dots, X_r, Y_1, Y_2, \dots, Y_{k-r}\}$ is optimal if it maximizes $ARD(\{X_1, X_2, \dots, X_r, Y_1, Y_2, \dots, Y_{k-r}\})$.

Example 1.
Revisiting the school classes assignment problem with students 4, 3, 2, and 1, $a_i = i$ for $i \in \{1, 2, 3, 4\}$, and $k = 2$, we calculate as follows.

For $\{Y_1, Y_2\} = \{\{4, 3\}, \{2, 1\}\}$: $ARD(\{Y_1, Y_2\}) = \frac{1}{2} \cdot 1 + \frac{1}{2} \cdot 1 = 1$;

For $\{Y_1, Y_2\} = \{\{4, 2\}, \{3, 1\}\}$: $ARD(\{Y_1, Y_2\}) = \frac{1}{2} \cdot 2 + \frac{1}{2} \cdot 2 = 2$;

For $\{Y_1, Y_2\} = \{\{4, 1\}, \{3, 2\}\}$: $ARD(\{Y_1, Y_2\}) = \frac{1}{2} \cdot 3 + \frac{1}{2} \cdot 1 = 2$.

Thus, whereas the latter two assignments are optimal, the first assignment is not optimal. We can already see that for a given set N and an ordered vector of performances that are independent of the performance of comparators $(a_1, a_2, \dots, a_n)$, more than one optimal division may exist.

Prior to solving the general assignment problem, two additional definitions will be of help.

Definition 4.
Assume that $N = \{1, 2, \dots, n\}$, $k < n$, $q = \lfloor \frac{n}{k} \rfloor$, and $n = kq + r$. The partition of N into k subsets, henceforth the k-partition, is a family of sets $\{A_1, A_2, \dots, A_{2q+1}\}$ such that for $l \in \{0, 1, \dots, q\}$

$$A_{2l+1} = \{lk + 1, lk + 2, \dots, lk + r\}$$

if $r \neq 0$ and

$$A_{2l+1} = \varnothing$$

if $r = 0$, and for $l \in \{1, \dots, q\}$

$$A_{2l} = \{(l-1)k + r + 1, (l-1)k + r + 2, \dots, lk\}.$$

Remark 1. For fixed N and k, the k-partition of N $\{A_1, A_2, \dots, A_{2q+1}\}$ is uniquely defined, $A_1, A_2, \dots, A_{2q+1}$ are pairwise disjoint, and $\bigcup_{i=1}^{2q+1} A_i = N$.

Remark 2. If $x \in A_i$, $y \in A_j$, and $i < j$, then $x < y$.

Definition 5.
We say that the division $\{X_1, X_2, \dots, X_r, Y_1, Y_2, \dots, Y_{k-r}\}$ of N is leveled if and only if for every $l \in \{0, 1, \dots, q\}$, $i \in \{1, 2, \dots, r\}$, and $j \in \{1, 2, \dots, k-r\}$

$$|X_i \cap A_{2l+1}| = 1, \; |Y_j \cap A_{2l+1}| = 0,$$

and for every $l \in \{1, \dots, q\}$, $i \in \{1, 2, \dots, r\}$, and $j \in \{1, 2, \dots, k-r\}$

$$|X_i \cap A_{2l}| = 0, \; |Y_j \cap A_{2l}| = 1,$$

where $\{A_1, A_2, \dots, A_{2q+1}\}$ is the k-partition of N.

Example 2.

(i) Revisiting once again the school classes assignment problem, $N = \{1, 2, 3, 4\}$ and $k = 2$, we have that $q = 2$ and that $r = 0$. Therefore, the *2*-partition of N is:

$$A_1 = A_3 = A_5 = \varnothing,$$

$$A_2 = \{1,2\},$$

$$A_4 = \{3,4\}.$$

The division $\{Y_1, Y_2\}$ of N is leveled if and only if each of the sets Y_i, $i = 1, 2$, contains exactly one element from A_2, and exactly one element from A_4. Namely the divisions {{4,2},{3,1}} and {{4,1},{3,2}} are leveled, whereas the division {{4,3},{2,1}} is not leveled.

(ii) Alternatively, let $N = \{1, 2, 3, 4, 5\}$ and $k = 2$, so that $q = 2$, $r = 1$, and the *2*-partition of N is:

$$A_1 = \{1\},$$

$$A_2 = \{2\},$$

$$A_3 = \{3\},$$

$$A_4 = \{4\},$$

$$A_5 = \{5\}.$$

Thus, there exists only one leveled division of N: $\{X_1, Y_1\} = \{\{1, 3, 5\}, \{2, 4\}\}$.

(iii) As yet another alternative, let $N = \{1, 2, \ldots, 18\}$ and $k = 5$, so we have that $q = 3$, and that $r = 3$, and the *5*-partition of N is:

$$A_1 = \{1,2,3\},$$

$$A_2 = \{4,5\},$$

$$A_3 = \{6,7,8\},$$

$$A_4 = \{9,10\},$$

$$A_5 = \{11,12,13\},$$

$$A_6 = \{14,15\},$$

$$A_7 = \{16,17,18\}.$$

The division $\{X_1, X_2, X_3, Y_1, Y_2\}$ of N is leveled if and only if each of the sets X_i, $i = 1, 2, 3$ consists of exactly four elements: one from A_1, one from A_3, one from A_5, and one from A_7, and each of the sets Y_i, $i = 1, 2$ consists of exactly three elements: one from A_2, one from A_4, and one from A_6. There are multiple leveled divisions of N. For example, the divisions {{18,12,8,1},{17,13,6,3},{16,11,7,2},{15,9,4},{14,10,5}} and {{18,13,8,3},{17,12,7,2},{16,11,6,1},{15,10,5},{14,9,4}} are leveled. It is easy to see that not all the divisions of N are leveled. For example, the division {{18,16,2,1},{15,10,6,3},{13,12,11,9},{17,14,4},{8,7,5}} is not leveled.

Lemma 1. Optimal divisions are leveled

Let $\{X_1, X_2, \ldots, X_r, Y_1, Y_2, \ldots, Y_{k-r}\}$ be a division of N. If this division is optimal, then it is leveled.

Proof. In Appendix A.

Lemma 2. Leveled divisions have the same ARD
Let $\{X_1, X_2, \ldots, X_r, Y_1, Y_2, \ldots, Y_{k-r}\}$ be a leveled division of N. Then,

$$ARD(\{X_1, X_2, \ldots, X_r, Y_1, Y_2, \ldots, Y_{k-r}\}) = \frac{1}{q+1}\sum_{l=1}^{q+1}\left[(2l-q-2)\sum_{i\in A_{2l-1}} a_i\right] + \frac{1}{q}\sum_{l=1}^{q}\left[(2l-q-1)\sum_{i\in A_{2l}} a_i\right],$$

where $\{A_1, A_2, \ldots, A_{2q+1}\}$ is the k-partition of N. In particular, for every leveled division of N, $ARD(\{X_1, X_2, \ldots, X_r, Y_1, Y_2, \ldots, Y_{k-r}\})$ is the same.

Proof. In Appendix A.

Claim 1. Characterization of the optimal division
Assume that $N = \{1, 2, \ldots, n\}$, $k<n$, $q = \lfloor \frac{n}{k} \rfloor$, and $n = kq + r$.

(a) A division of N $\{X_1, X_2, \ldots, X_r, Y_1, Y_2, \ldots, Y_{k-r}\}$ is optimal if and only if it is leveled.
(b) There are $(r!)^q((k-r)!)^{q-1}$ optimal divisions of N.

Proof. In Appendix A.

Remark 3. If $k = 2$ and n is odd, then $r = k - r = 1$, $(r!)^q((k-r)!)^{q-1} = 1$, and there is only one optimal division of N: $\{X_1, Y_1\}$ where $X_1 = \{1, 3, \ldots, n\}$ is the group of odd-numbered individuals, and $Y_1 = \{2, 4, \ldots, n-1\}$ is the group of even-numbered individuals. For any other combination of n and k, there are always several optimal divisions.

For instance, in Example 2 (ii) and as already noted, there is only one optimal division: $\{\{5, 3, 1\}, \{4, 2\}\}$. However, if $k = 2$ and n is even, then $r = 0$, $q = \frac{n}{2}$, and there are $(r!)^q((k-r)!)^{q-1} = 2^{\frac{n}{2}-1}$ optimal divisions of N. In particular, in Example 2 (i) there are $2^{\frac{4}{2}-1} = 2$ optimal divisions: {{4,2},{3,1}} and {{4,1},{3,2}}. And in Example 2 (iii), where $n = 18$, $k = 5$, and $q = r = 3$, there are $(3!)^3(2!)^2 = 864$ optimal divisions.

It is of interest to add that the optimal solution to the maximization problem of ARD does not depend on the vector of performances that are independent of the performance of comparators $(a_1, a_2, \ldots, a_n)$; the optimal solution is premised on the feature that the performances that are independent of the performance of comparators are arranged in an ascending order (namely $a_1 < a_2 < \ldots < a_n$). To illustrate once again: the optimal divisions of $N = \{1, 2, 3, 4\}$ into $k = 2$ sets will be the same regardless of whether $(a_1, a_2, a_3, a_4) = (1, 2, 3, 4)$, whether $(a_1, a_2, a_3, a_4) = (1, 2, 4, 10)$, or whether $(a_1, a_2, a_3, a_4) = (1, 7, 9, 10)$. It is *the hierarchical order* that matters, rather than the cardinal values of the performances that are independent of the performance of comparators.[3]

3. Conclusions

We have studied how to divide a group of individuals into subgroups so as to maximally influence their performance in conditions of pressure exerted by the performance of comparators. For each population of size $n \in \mathbb{N}$, $n \geq 4$, we identified the set of divisions that maximizes aggregate pressure. For each n, the solution depends only on the ordinally measured performances that are independent of the performance of comparators.

Our analysis is based on several implicit assumptions, and thus has its limitations. For example, we assume that when individuals are assigned into groups, they have no better alternative options. That being said, we formulate a rule of assignment that was not presented before. Follow-up research could

[3]Quite obviously, the maximal value of $ARD(\{X_1, X_2\})$ depends on $(a_1, a_2, \ldots, a_n)$. As calculated in Example 1, for $N = 4$ and $(a_1, a_2, a_3, a_4) = (1, 2, 3, 4)$, the maximal value of $ARD(\{X_1, X_2\})$ is 2. For $N = 4$ and $(a_1, a_2, a_3, a_4) = (1, 2, 4, 10)$, the maximal value of $ARD(A, B)$ is 5.5. Nonetheless, in both cases, the *set* of optimal divisions $\{\{\{4, 1\}, \{3, 2\}\}, \{\{4, 2\}, \{3, 1\}\}\}$ is the same.

build on our framework, looking into issues of robustness and, perhaps particularly rewarding, put our claims to laboratory and empirical tests.

Some literature maintains that comparisons are with worse-off individuals, and not – as we have assumed – with better-off individuals. Although we believe that the weight of the evidence supports our stance, we note that studies (such as Boyce, Brown, & Moore, 2010) which looked at both effects found strong support that comparisons with better-off individuals are substantially more important than comparisons with worse-off individuals. To the extent that comparisons could be both ways while those with the better-off individuals dominate, then in this regard our setting is a "limit" case.

In closing, it is interesting to note that the behavior that we modeled in this paper is not the only behavior that a group of workers or students can exhibit. For example, in the domain of workers and management, rather than exerting effort to move up in the performance hierarchy, a group of workers may exercise social control to hold performance at a low level. The reason for this is fear that, otherwise, management may set as a standard a level of effort that is too high. The possibility of such behavior was documented a long time ago in the classical study of Roethlisberger and Dickson (1939). In our setting we did not allow strategic behavior of this type. The assignment problem that we modeled is based on a management's drive to maximize the aggregate pressure of workers to perform better, where what propels that behavior is an unimpeded desire of each worker to curtail his relative deprivation. The distinction between the case studied by Roethlisberger and Dickson and our setting notwithstanding, what is common to their study and to our study is that their years of research led them (citing from an abstract of their study) "to a critical evaluation of the traditional view that workers … [can] be considered apart from their social setting and treated as essentially 'economic men.'"

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Appendix A. Proofs

For ease of reference, we use the following notation: if $\{X_1, X_2, \ldots, X_r, Y_1, Y_2, \ldots, Y_{k-r}\}$ is a division of N, then for each $i \in \{1, 2, \ldots, r\}$ we denote the comparison-free performances of the individuals from the set X_i as $x_i^1, x_i^2, \ldots, x_i^{q+1}$, where $x_i^t < x_i^u$, for $t, u \in \{1, 2, \ldots, q+1\}$, if and only if $t < u$. For each $j \in \{1, 2, \ldots, k-r\}$ we denote the comparison-free performances of the individuals from the set Y_j as $y_j^1, y_j^2, \ldots, y_j^q$, where $y_i^t < y_i^u$, for $t, u \in \{1, 2, \ldots, q\}$, if and only if $t < u$. Moreover, for $Z \subset N$ and $z \in Z$, we denote $Z(z) = \{i \in Z : i \leq z\}$ and $n(Z, z) = |Z(z)|$ (namely if $Z = \{z_1, z_2, \ldots, z_p\}$ and $z_1 < z_2 < \ldots < z_p$, then $n(Z, z_i) = i$).

Prior to proving Lemma 1, Lemma 2, and Claim 1, we present a supportive lemma that yields a helpful auxiliary result.

Supportive Lemma.

Let $s \leq n$ and let $\varphi_S : \{1, 2, \ldots, s\} \to N$ be an increasing injection. If $S = \{\varphi_S(1), \varphi_S(2), \ldots, \varphi_S(s)\}$, then

$$\sum_{m \in S} RD_S(m) = \frac{1}{s}\Big((s-1)a_{\varphi_S(s)} + (s-3)a_{\varphi_S(s-1)} + \ldots + (-s+1)a_{\varphi_S(1)}\Big) = \frac{1}{s}\sum_{l=1}^{s}(2l-s-1)a_{\varphi_S(l)}.$$

In particular, if $\{X_1, X_2, \ldots, X_r, Y_1, Y_2, \ldots, Y_{k-r}\}$ is a division of N, then for $i \in \{1, 2, \ldots, r\}$

$$\sum_{m \in X_i} RD_{X_i}(m) = \frac{1}{q+1}\sum_{l=1}^{q+1}(2l-q-2)x_i^l,$$

and for $j \in \{1, 2, \ldots, k-r\}$

$$\sum_{m \in Y_j} RD_{Y_j}(m) = \frac{1}{q}\sum_{l=1}^{q}(2l-q-1)y_j^l.$$

Proof of the Supportive Lemma.

By Definition 2 and the definition of the set S

$$\sum_{m\in S} RD_S(m) = \frac{1}{|S|}\sum_{m,l\in S}\max\{a_m - a_l, 0\} = \frac{1}{s}\sum_{l=1}^{s}\sum_{m=l+1}^{s}\left(a_{\varphi_S(m)} - a_{\varphi_S(l)}\right)$$

$$= \frac{1}{s}\sum_{l=1}^{s}\left[\left(\sum_{m=l+1}^{s} a_{\varphi_S(m)}\right) - (s-l)a_{\varphi_S(l)}\right] = \frac{1}{s}\left(\sum_{l=1}^{s}\sum_{m=i+1}^{s} a_{\varphi_S(m)} - \sum_{l=1}^{s}(s-l)a_{\varphi_S(l)}\right)$$

$$= \frac{1}{s}\left(\sum_{l=1}^{s}(l-1)a_{\varphi_S(l)} - \sum_{l=1}^{s}(s-l)a_{\varphi_S(l)}\right) = \frac{1}{s}\sum_{i=1}^{s}(2l-s-1)a_{\varphi_S(l)}.$$

In particular, for $S = X_i$, $i \in \{1, 2, \ldots, r\}$, it holds that $s = q+1$ and that $a_{\varphi_S(l)} = x_i^l$, thus

$$\sum_{m\in X_i} RD_{X_i}(m) = \frac{1}{q+1}\sum_{l=1}^{q+1}(2l-q-2)x_i^l,$$

and for $S = Y_j$, $j \in \{1, 2, \ldots, k-r\}$, it holds that $s = q$ and that $a_{\varphi_S(l)} = y_j^l$, thus

$$\sum_{m\in Y_j} RD_{Y_j}(m) = \frac{1}{q}\sum_{l=1}^{q}(2l-q-1)y_j^l.$$

Q.E.D.

Proof of Lemma 1.

To show that a division of N $\{X_1, X_2, X_3 \ldots, X_r, Y_1, Y_2, \ldots, Y_{k-r}\}$ that is not leveled is not optimal, we note that, then, at least one of the following four cases occurs.
(i) there exists $l \in \{1, 2, \ldots, 2q+1\}$ such that l is odd and there exists $i \in \{1, 2, \ldots, r\}$ where $|X_i \cap A_l| \neq 1$;
(ii) there exists $l \in \{1, 2, \ldots, 2q+1\}$ such that l is odd and there exists $j \in \{1, 2, \ldots, k-r\}$ where $|Y_j \cap A_l| \neq 0$;
(iii) there exists $l \in \{1, 2, \ldots, 2q+1\}$ such that l is even and there exists $i \in \{1, 2, \ldots, r\}$ where $|X_i \cap A_l| \neq 0$;
(iv) there exists $l \in \{1, 2, \ldots, 2q+1\}$ such that l is even and there exists $j \in \{1, 2, \ldots, k-r\}$ where $|Y_j \cap A_l| \neq 1$.

Assuming that either (i) or (ii) occurs for $l \in \{1, 2, \ldots, 2q+1\}$ and $X_i \cap A_l \neq \emptyset$, namely that $|X_i \cap A_l| \geq 1$ for each $i \in \{1, 2, \ldots, r\}$, then in case (i)

$$|A_l| \geq \left|\bigcup_{i=1}^{r}(A_l \cap X_i)\right| = \sum_{i=1}^{r}|A_l \cap X_i| > r,$$

and in case (ii)

$$|A_l| \geq \left|\bigcup_{i=1}^{r}(A_l \cap X_i) \cup (A_l \cap Y_j)\right| = \sum_{i=1}^{r}|A_l \cap X_i| + |A_l \cap Y_j| \geq r+1 > r.$$

In both cases, we reach a contradiction with $|A_l| = r$. Therefore, if (i) or if (ii) occurs, then there exists $i \in \{1, 2, \ldots, r\}$ such that $X_i \cap A_l = \emptyset$. Analogously, we can show that if (iii) or if (iv) occurs, then there exists $j \in \{1, 2, \ldots, k-r\}$ where $Y_j \cap A_l = \emptyset$.

Thus, there exists $l \in \{1, 2, \ldots, 2q+1\}$ such that either l is odd and there exists $i \in \{1, 2, \ldots, r\}$ where $X_i \cap A_l = \emptyset$, or l is even and there exists $j \in \{1, 2, \ldots, k-r\}$ where $Y_j \cap A_l = \emptyset$. Henceforth, such l is termed *level-breaking*. We define

$$\lambda \equiv \max\{l \in \{1, 2, \ldots, 2q+1\} : \ l \text{ is level-breaking}\}$$

We note that as long as $\{X_1, X_2, X_3 \ldots, X_r, Y_1, Y_2, \ldots, Y_{k-r}\}$ is not leveled, $\lambda \in \{1, 2, \ldots, 2q+1\}$ is well defined. By definition of a leveled division and Remark 2, if $\lambda < 2l-1 \leq 2q+1$, then for every $i \in \{1, 2, \ldots, r\}$, $x_i^l \in A_{2l-1}$. Also, if $\lambda < 2l \leq 2q$, then for every $j \in \{1, 2, \ldots, k-r\}$, $y_j^l \in A_{2l}$.

We consider first the case of an odd λ: $\lambda = 2l_0 - 1$. In this case, there exists $i \in \{1, 2, \ldots, r\}$ such that $X_i \cap A_\lambda = \emptyset$ and either

(a) there exists $j \in \{1, 2, \ldots, r\}, \{i\}$ where $|X_j \cap A_\lambda| > 1$,
or

(b) there exists $j \in \{1, 2, \ldots, k-r\}$ where $Y_j \cap A_\lambda \neq \emptyset$.

Without loss of generality and for simplicity's sake, we assume that $i = 1$.

Considering sub-case (a), we assume that there exists $j \in \{1, 2, \ldots, r\} \setminus \{i\}$ such that $|X_j \cap A_\lambda| > 1$. Again, without loss of generality and for simplicity's sake, we assume that $j = 2$. Let $x = \min(X_2 \cap A_\lambda)$. Then $n(X_2, x) < l_0$, because there are at least $q + 2 - l_0$ individuals in X_2 whose comparison-free performance is better than x ($q + 1 - l_0$ of them belong, respectively, to sets $A_{2l_0+1}, A_{2l_0+3}, \ldots, A_{2q+1}$, and at least one other individual from X_2 belongs to $X_2 \cap A_\lambda$ and performs better). We define

$$X_1' \equiv X_1 \cup \{x\} \setminus \{x_1^{l_0}\};\ X_2' = X_2 \cup \{x_1^{l_0}\} \setminus \{x\}.$$

Then $\{X_1', X_2', X_3 \ldots, X_r, Y_1, Y_2, \ldots, Y_{k-r}\}$ is also a division, and

$$\begin{aligned} &ARD(\{X_1', X_2', X_3 \ldots, X_r, Y_1, Y_2, \ldots, Y_{k-r}\}) - ARD(\{X_1, X_2, X_3 \ldots, X_r, Y_1, Y_2, \ldots, Y_{k-r}\}) \\ &= \sum_{S \in \{X_1', X_2', \ldots, X_r, Y_1, Y_2, \ldots, Y_{k-r}\}} \sum_{i \in S} RD_S(i) - \sum_{S \in \{X_1, X_2, \ldots, X_r, Y_1, Y_2, \ldots, Y_{k-r}\}} \sum_{i \in S} RD_S(i) \\ &= \left(\sum_{i \in X_1'} RD_{X_1'}(i) + \sum_{i \in X_2'} RD_{X_2'}(i) \right) - \left(\sum_{i \in X_1} RD_{X_1}(i) + \sum_{i \in X_2} RD_{X_2}(i) \right) \\ &= \sum_{i \in X_1'} RD_{X_1'}(i) - \sum_{i \in X_1} RD_{X_1}(i) + \sum_{i \in X_2'} RD_{X_2'}(i) - \sum_{i \in X_2} RD_{X_2}(i). \end{aligned}$$

From Remark 2 and the fact that $x_1^{l_0} \notin A_l$ for $l \geq \lambda$, it follows that $x > x_1^{l_0}$. Also, $x_1^l \in A_{2l-1}$ for $l > l_0$, and $x \in A_\lambda = A_{2l_0-1}$ so, therefore, $n(X_1, x_1^{l_0}) = n(X_1', x) = l_0$. By the Supportive Lemma, we obtain that

$$\sum_{i \in X_1} RD_{X_1}(i) = \frac{1}{q+1} \sum_{l=1}^{q+1} (2l - q - 2) x_1^l,$$

and that

$$\sum_{i \in X_1'} RD_{X_1'}(i) = \frac{1}{q+1} \sum_{l=1}^{q+1} (2l - q - 2) x_1^l - \frac{1}{q+1} (2l_0 - q - 2) x_1^{l_0} + \frac{1}{q+1} (2l_0 - q - 2) x.$$

Therefore,

$$\sum_{i \in X_1'} RD_{X_1'}(i) - \sum_{i \in X_1} RD_{X_1}(i) = \frac{1}{q+1} (2l_0 - q - 2)(x - x_1^{l_0}).$$

Moreover,

$$\sum_{i \in X_2} RD_{X_2}(i) = \frac{1}{q+1} \sum_{l=1}^{q+1} (2l - q - 2) x_2^l.$$

We know that $x > x_1^{l_0}$, thus $n(X_2', x_1^{l_0}) \leq n(X_2, x) < l_0$. Assume that $n(X_2, x) = l_1 < l_0$ (namely $x = x_2^{l_1}$) and that $n(X_2', x_1^{l_0}) = l_2 \leq l_1$. Then

$$\sum_{i \in X_2'} RD_{X_2'}(i) = \frac{1}{q+1} \left[\sum_{l=l_1+1}^{q+1} (2l - q - 2) x_2^l + \sum_{l=l_2}^{l_1-1} (2l - q) x_2^l + (2l_2 - q - 2) x_1^{l_0} + \sum_{l=1}^{l_2-1} (2l - q - 2) x_2^l \right].$$

Thus,

$$\sum_{i \in X_2'} RD_{X_2'}(i) - \sum_{i \in X_2} RD_{X_2}(i) = \frac{1}{q+1} \left[\sum_{l=l_2}^{l_1-1} 2x_2^l + (2l_2 - q - 2) x_1^{l_0} - (2l_1 - q - 2) x \right].$$

Finally,

$$ARD(\{X_1', X_2', X_3 \ldots, X_r, Y_1, Y_2, \ldots, Y_{k-r}\}) - ARD(\{X_1, X_2, X_3 \ldots, X_r, Y_1, Y_2, \ldots, Y_{k-r}\})$$

$$= \sum_{i \in X_1'} RD_{X_1'}(i) - \sum_{i \in X_1} RD_{X_1}(i) + \sum_{i \in X_2'} RD_{X_2'}(i) - \sum_{i \in X_2} RD_{X_2}(i)$$

$$= \frac{1}{q+1}\left[(2l_0 - q - 2)(x - x_1^{l_0}) + \sum_{l=l_2}^{l_1-1} 2x_2^l + (2l_2 - q - 2)x_1^{l_0} - (2l_1 - q - 2)x\right]$$

$$= \frac{1}{q+1}\left[2(l_0 - l_1)x - 2(l_0 - l_2)x_1^{l_0} + \sum_{l=l_2}^{l_1-1} 2x_2^l\right] \geq \frac{2(l_0 - l_1)}{q+1}(x - x_1^{l_0}) > 0.$$

Thus,

$$ARD(\{X_1', X_2', X_3 \ldots, X_r, Y_1, Y_2, \ldots, Y_{k-r}\}) > ARD(\{X_1, X_2, X_3 \ldots, X_r, Y_1, Y_2, \ldots, Y_{k-r}\}).$$

Considering now sub-case (b), we assume that there exists $j \in \{1, 2, \ldots, k-r\}$ such that $Y_j \cap A_\lambda \neq \emptyset$. Without loss of generality and for simplicity's sake, we assume that $j = 1$. Let $x = \min(Y_1 \cap A_\lambda)$. Then $n(Y_1, x) \leq l_0 - 1$ because there are at least $q - l_0 + 1$ individuals in Y_1 whose comparison-free performance is better than x (each of the sets $A_{2l_0}, A_{2l_0+2}, A_{2l_0+4}, \ldots, A_{2q}$ contains exactly one of these individuals and there can be individuals whose comparison-free performance is better than x in $Y_1 \cap A_\lambda$). We define

$$X_1' \equiv X_1 \cup \{x\} \setminus \{x_1^{l_0}\};\ Y_1' = Y_1 \cup \{x_1^{l_0}\} \setminus \{x\}.$$

Then $\{X_1', X_2, X_3 \ldots, X_r, Y_1', Y_2, \ldots, Y_{k-r}\}$ is also a division, and

$$\begin{aligned} &ARD(\{X_1', X_2, X_3 \ldots, X_r, Y_1', Y_2, \ldots, Y_{k-r}\}) - ARD(\{X_1, X_2, X_3 \ldots, X_r, Y_1, Y_2, \ldots, Y_{k-r}\}) \\ &= \sum_{S \in \{X_1', X_2, \ldots, X_r, Y_1', Y_2, \ldots, Y_{k-r}\}} \sum_{i \in S} RD_S(i) - \sum_{S \in \{X_1, X_2, \ldots, X_r, Y_1, Y_2, \ldots, Y_{k-r}\}} \sum_{i \in S} RD_S(i) \\ &= \left(\sum_{i \in X_1'} RD_{X_1'}(i) + \sum_{i \in Y_1'} RD_{Y_1'}(i)\right) - \left(\sum_{i \in X_1} RD_{X_1}(i) + \sum_{i \in Y_1} RD_{Y_1}(i)\right) \\ &= \sum_{i \in X_1'} RD_{X_1'}(i) - \sum_{i \in X_1} RD_{X_1}(i) + \sum_{i \in Y_1'} RD_{Y_1'}(i) - \sum_{i \in Y_1} RD_{Y_1}(i). \end{aligned}$$

Identically, as in the sub-case (a) we obtain that

$$\sum_{i \in X_1'} RD_{X_1'}(i) - \sum_{i \in X_1} RD_{X_1}(i) = \frac{1}{q+1}(2l_0 - q - 2)(x - x_1^{l_0}).$$

Moreover,

$$\sum_{i \in Y_1} RD_{Y_1}(i) = \frac{1}{q}\sum_{l=1}^{q}(2l - q - 1)y_1^l.$$

We know that $x > x_1^{l_0}$, thus $n(Y_1', x_1^{l_0}) \leq n(Y_1, x) \leq l_0 - 1 < l_0$. Assume that $n(Y_1, x) = l_1 < l_0$ (namely $x = y_1^{l_1}$) and that $n(X_1', x_1^{l_0}) = l_2 \leq l_1$. Then

$$\sum_{i \in Y_1'} RD_{Y_1'}(i) = \frac{1}{q}\left[\sum_{l=l_1+1}^{q+1}(2l - q - 1)y_1^l + \sum_{l=l_2}^{l_1-1}(2l - q + 1)y_1^l + (2l_2 - q - 1)x_1^{l_0} + \sum_{l=1}^{l_2-1}(2l - q - 1)y_1^l\right].$$

Thus,

$$\sum_{i \in Y_1'} RD_{Y_1'}(i) - \sum_{i \in Y_1} RD_{Y_1}(i) = \frac{1}{q}\left[\sum_{l=l_2}^{l_1-1} 2y_1^l + (2l_2 - q - 1)x_1^{l_0} - (2l_0 - q - 3)x\right].$$

Finally,

$$ARD(\{X_1{}', X_2, X_3 \ldots, X_r, Y_1{}', Y_2, \ldots, Y_{k-r}\}) - ARD(\{X_1, X_2, X_3 \ldots, X_r, Y_1, Y_2, \ldots, Y_{k-r}\})$$

$$= \sum_{i \in X_1{}'} RD_{X_1{}'}(i) - \sum_{i \in X_1} RD_{X_1}(i) + \sum_{i \in Y_1{}'} RD_{Y_1{}'}(i) - \sum_{i \in Y_1} RD_{Y_1}(i)$$

$$= \frac{1}{q+1}(2l_0 - q - 2)(x - x_1^{l_0}) + \frac{1}{q}\left[\sum_{l=l_2}^{l_1-1} 2y_1^l + (2l_2 - q - 1)x_1^{l_0} - (2l_0 - q - 3)x\right]$$

$$\geq \frac{1}{q}\left[(2l_0 - q - 3)(x - x_1^{l_0}) + \sum_{l=l_2}^{l_1-1} 2y_1^l + (2l_2 - q - 1)x_1^{l_0} - (2l_0 - q - 3)x\right] \geq 2x_1^{l_0} > 0.$$

Thus,

$$ARD(\{X_1{}', X_2{}', X_3 \ldots, X_r, Y_1, Y_2, \ldots, Y_{k-r}\}) > ARD(\{X_1, X_2, X_3 \ldots, X_r, Y_1, Y_2, \ldots, Y_{k-r}\}).$$

For both sub-cases (a) and (b) we obtained that there exists a division which yields a higher ARD than $\{X_1, X_2, X_3 \ldots, X_r, Y_1, Y_2, \ldots, Y_{k-r}\}$, so if λ is odd, then $\{X_1, X_2, X_3 \ldots, X_r, Y_1, Y_2, \ldots, Y_{k-r}\}$ is not optimal. Analogously, we can obtain the same type of result when λ is even. The proof of this case, which is similar to the proof for an odd λ, is available from the authors on request.

To sum up, for every $\lambda \in \{1, 2, \ldots, 2q+1\}$, $\{X_1, X_2, X_3 \ldots, X_r, Y_1, Y_2, \ldots, Y_{k-r}\}$ is not optimal. Therefore, if $\{X_1, X_2, X_3 \ldots, X_r, Y_1, Y_2, \ldots, Y_{k-r}\}$ is optimal, then λ cannot exist and, consequently, $\{X_1, X_2, X_3 \ldots, X_r, Y_1, Y_2, \ldots, Y_{k-r}\}$ is leveled. Q.E.D.

Proof of Lemma 2.

We assume that $\{X_1, X_2, \ldots, X_r, Y_1, Y_2, \ldots, Y_{k-r}\}$ is a leveled division of N. First, we note that for every $i \in \{1, 2, \ldots, r\}$ and for every $l \in \{1, 2, \ldots, q+1\}$, $x_i^l \in A_{2l-1}$. Also, for every $j \in \{1, 2, \ldots, k-r\}$ and for every $l \in \{1, 2, \ldots, q\}$, $y_j^l \in A_{2l}$. Therefore, for every $l \in \{1, 2, \ldots, q+1\}$

$$\sum_{i=1}^{r} x_i^l = \sum_{i \in A_{2l-1}} a_i,$$

and for every $l \in \{1, 2, \ldots, q\}$

$$\sum_{i=1}^{k-r} y_i^l = \sum_{i \in A_{2l}} a_i.$$

Thus, by the Supportive Lemma

$$\sum_{i=1}^{r} \sum_{m \in X_i} RD_{X_i}(m) = \sum_{i=1}^{r} \frac{1}{q+1} \sum_{l=1}^{q+1} (2l - q - 2)x_i^l = \frac{1}{q+1} \sum_{l=1}^{q+1} (2l - q - 2) \sum_{i=1}^{r} x_i^l = \frac{1}{q+1} \sum_{l=1}^{q+1} \left[(2l - q - 2) \sum_{i \in A_{2l-1}} a_i\right],$$

and

$$\sum_{i=1}^{k-r} \sum_{m \in Y_i} RD_{Y_i}(m) = \sum_{i=1}^{r} \frac{1}{q} \sum_{l=1}^{q} (2l-q-1)y_i^l = \frac{1}{q} \sum_{l=1}^{q} (2l-q-1) \sum_{i=1}^{k-r} y_i^l = \frac{1}{q} \sum_{l=1}^{q} \left[(2l-q-1) \sum_{i \in A_{2l}} a_i\right].$$

Finally,

$$ARD(\{X_1, X_2, \ldots, X_r, Y_1, Y_2, \ldots, Y_{k-r}\}) = \sum_{S \in \{X_1, X_2, \ldots, X_r, Y_1, Y_2, \ldots, Y_{k-r}\}} \sum_{m \in S} RD_S(m)$$

$$= \sum_{i=1}^{r} \sum_{m \in X_i} RD_{X_i}(m) + \sum_{i=1}^{k-r} \sum_{m \in Y_i} RD_{Y_i}(m) = \frac{1}{q+1} \sum_{l=1}^{q+1} \left[(2l-q-2) \sum_{i \in A_{2l-1}} a_i\right] + \frac{1}{q} \sum_{l=1}^{q} \left[(2l-q-1) \sum_{i \in A_{2l}} a_i\right].$$

Q.E.D.

Proof of Claim 1.

Proof of part (a)

By Lemma 1 we know that every optimal division is leveled. Because the set of all possible different divisions of N is finite, at least one optimal division exists, and it is leveled. Because by Lemma 2 all leveled divisions of N yield the same ARD, then all leveled divisions of N yield the same ARD as the optimal division. Therefore, every leveled division is optimal.

Proof of part (b)

Because the set of the optimal divisions of N is equal to the set of the leveled divisions of N, it is sufficient to calculate the number of leveled divisions of N.

We first calculate the number of sequences $(X_1, X_2, \ldots, X_r)$ for which a family of subsets $\{Y_1, Y_2, \ldots, Y_{k-r}\}$ exists such that $\{X_1, X_2, \ldots, X_r, Y_1, Y_2, \ldots, Y_{k-r}\}$ is a leveled division of N. Each of the sets X_i, $i = 1, 2, \ldots, r$ consists of one element from each of the sets $\{A_1, A_3, \ldots, A_{2q+1}\}$, where $\{A_1, A_2, \ldots, A_{2q+1}\}$ is the k-partition of N. For $j = 0, 1, \ldots, q$, $|A_{2j+1}| = r$. Therefore, each sequence $(X_1, X_2, \ldots, X_r)$ is equivalent to one permutation of the sets $A_1, A_3, \ldots, A_{2q+1}$,[4] and $(r!)^{q+1}$ such permutations exist.

To calculate the number of unordered families $\{X_1, X_2, \ldots, X_r\}$ for which there exists a family of sets $\{Y_1, Y_2, \ldots, Y_{k-r}\}$ such that $\{X_1, X_2, \ldots, X_r, Y_1, Y_2, \ldots, Y_{k-r}\}$ is a leveled division of N, we need to divide the number of such sequences $(X_1, X_2, \ldots, X_r)$ by the number of permutations of the sequence $(X_1, X_2, \ldots, X_r)$. We obtain

$$\frac{(r!)^{q+1}}{r!} = (r!)^q.$$

Analogously, we can calculate the number of unordered families $\{Y_1, Y_2, \ldots, Y_{k-r}\}$ for which a family of sets $\{X_1, X_2, \ldots, X_r\}$ exists such that $\{X_1, X_2, \ldots, X_r, Y_1, Y_2, \ldots, Y_{k-r}\}$ is a leveled division of N, and we obtain $((k-r)!)^{q-1}$.

We note that $\{X_1, X_2, \ldots, X_r, Y_1, Y_2, \ldots, Y_{k-r}\}$ is a leveled division of N if and only if a family of sets $\{Y_1', Y_2', \ldots, Y_{k-r}'\}$ exists for $\{X_1, X_2, \ldots, X_r\}$ such that $\{X_1, X_2, \ldots, X_r, Y_1', Y_2', \ldots, Y_{k-r}'\}$ is a leveled division of N, and a family of sets $\{X_1', X_2', \ldots, X_r'\}$ exists for $\{Y_1, Y_2, \ldots, Y_{k-r}\}$ such that $\{X_1', X_2', \ldots, X_r', Y_1, Y_2, \ldots, Y_{k-r}\}$ is a leveled division of N. Moreover, because the distribution of the set $A_1 \cup A_3 \cup \ldots \cup A_{2q+1}$ into the family of sets $\{X_1, X_2, \ldots, X_r\}$ is independent of the distribution of the set $A_2 \cup A_4 \cup \ldots \cup A_{2q}$ into the family of sets $\{Y_1, Y_2, \ldots, Y_{k-r}\}$, the number of leveled divisions of N is $(r!)^q((k-r)!)^{q-1}$. Q.E.D.

Appendix B. The measure of relative deprivation in Definition 2

B1. A concise historical account of the "adoption" of the sociological-psychological concept of relative deprivation by the discipline of economics

A considerable amount of economic analysis has been inspired by the sociological-psychological concepts of relative deprivation (*RD*) and reference (comparison) groups.[5] Economists have come to consider these concepts as appropriate tools for studying comparisons that affect an individual's behavior, and - in particular - comparisons with related individuals whose incomes are higher than his own income (consult the large literature spanning from Duesenberry, 1949, to, for example, Clark, Frijters, & Shields, 2008). An individual has an unpleasant sense of being relatively deprived when he lacks a desired good and perceives that others in his reference group possess that good (Runciman, 1966). Given the income distribution of the individual's reference group, the individual's *RD* is the sum of the deprivation caused by every income unit that he lacks (Yitzhaki, 1979; Ebert & Moyes, 2000; Stark, Bielawski, & Falniowski, 2017).

The pioneering study in modern times that opened the way to research on RD and primary (reference) groups is the 1949 two-volume set of Stouffer et al. *Studies in Social Psychology in World War II: The American Soldier*. That work documented the distress caused not by a low military rank and weak prospects of promotion (military police) but rather by the faster pace of promotion of others (air force). It also documented the lesser dissatisfaction of black soldiers stationed in the South who compared themselves with black civilians in the South than the dissatisfaction of their counterparts stationed in the North who compared themselves with black civilians in the North. Stouffer's research was followed by a large social-psychological literature. Economics has caught up relatively late, and only partially. This is rather surprising because eminent economists in the past understood well that people compare themselves to others around them, and that social comparisons are of paramount importance for individuals' happiness, motivation, and actions. Even Adam Smith (1776) pointed to the social aspects of the necessities of life, and stressed the relative nature of poverty: "A linen shirt, for example, is, strictly speaking, not a necessary of life. The Greeks and Romans lived, I suppose, very comfortably, though they had no linen. But in the present times, through the greater part of Europe, a creditable day-laborer would be ashamed to appear in public without a linen shirt, the

[4] By a permutation of the sets $Z_1, Z_2, \ldots, Z_m$ we mean here a sequence of permutations $(\sigma_1, \sigma_2, \ldots, \sigma_m)$ such that σ_i is a permutation of Z_i for $i = 1, 2, \ldots, m$.

[5] The reference (comparison) group of an individual is the set of individuals with whom the individual naturally compares himself. (Consult Runciman, 1966; Singer, 1981.)

want of which would be supposed to denote that disgraceful degree of poverty [...]" (p. 465). Marx's (1849) observations that "Our wants and pleasures have their origin in the society; [... and] they are of a relative nature" (p. 33) emphasize the social nature of utility and the impact of an individual's relative position on his satisfaction. Inter alia, Marx wrote: "A house may be large or small; as long as the surrounding houses are equally small, it satisfies all social demands for a dwelling. But if a palace arises beside the little house, the house shrinks into a hut" (p. 33). Samuelson (1973), one of the founders of modern neoclassical economics, pointed out that an individual's utility does not depend only on what he consumes in *absolute* terms: "Because man is a social animal, what he regards as 'necessary comforts of life' depends on what he sees others consuming" (p. 218).

The relative income hypothesis, formulated by Duesenberry (1949), posits an asymmetry in the comparisons of income which affect the individual's behavior: the individual looks upward when making comparisons. Veblen's (1899) concept of pecuniary emulation explains why the behavior of an individual can be influenced by comparisons with the incomes of those who are richer. Because income determines the level of consumption, higher income levels may be the focus for emulation. Thus, an individual's income aspirations (to obtain the income levels of other individuals whose incomes are higher than his own) are shaped by the perceived consumption standards of the richer individuals. In that way, invidious comparisons affect behavior, that is, behavior which leads to "the achievement of a favourable comparison with other men [...]" (Veblen, 1899, p. 33).[6]

B2. Construction of the index of relative deprivation

Several recent insightful studies in social psychology (for example, Callan, Shead, & Olson, 2011; Smith, Pettigrew, Pippin, & Bialosiewicz, 2012) document how sensing *RD* impacts negatively on personal wellbeing, but these studies do not provide a method of calibrating it; a sign is not a magnitude. For the purpose of constructing a measure, a natural starting point is the work of Runciman (1966), who, as already noted in the preceding section, argued that an individual has an unpleasant sense of being relatively deprived when he lacks a desired good and perceives that others with whom he naturally compares himself possess that good. Runciman (1966, p. 19) writes as follows: "The more people a man sees promoted when he is not promoted himself, the more people he may compare himself with in a situation where the comparison will make him feel deprived," thus implying that the deprivation of not having, say, income y is an increasing function of the fraction of people in the individual's reference group who have y. To aid intuition and for the sake of concreteness, we resort to income-based comparisons, namely an individual feels relatively deprived when others in his reference group earn more than he does. It is assumed implicitly here that the earnings of others are publicly known. Alternatively, we can think of consumption, which might be more publicly visible than income, although these two variables can reasonably be assumed to be strongly positively correlated.

As an illustration of the relationship between the fraction of people possessing income y and the deprivation of an individual lacking y, consider a population (reference group) of six individuals with incomes {1,2,6,6,6,8}. Imagine a furniture store that in three distinct departments sells chairs, armchairs, and sofas. An income of 2 allows you to buy a chair. To be able to buy an armchair, you need an income that is a little bit higher than 2. To buy any sofa, you need an income that is a little bit higher than 6. Thus, when you go to the store and your income is 2, what are you "deprived of?" The answer is "of armchairs" and "of sofas." Mathematically, this deprivation can be represented by $P(Y>2)(6-2)+P(Y>6)(8-6)$, where $P(Y>a)$ stands for the fraction of those in the population whose income is higher than a, for $a=2,6$. The reason for this representation is that when you have an income of 2, you cannot afford anything in the department that sells armchairs, and you cannot afford anything in the department that sells sofas. Because not all those who are to your right in the ascendingly ordered income distribution can afford to buy a sofa, but they can all afford to buy armchairs, a breakdown into the two (weighted) terms $P(Y>2)(6-2)$ and $P(Y>6)(8-6)$ is needed. This way, we get to the very essence of the measure of *RD* presented in this paper: we take into account the fraction of the reference group (population) who possess some good which you do not, and we weigh this fraction by the "excess value" of that good. Because income enables an individual to afford to consume certain goods, we refer to comparisons based on income.

Formally, let $a=(a_1,\ldots,a_m)$ be the vector of incomes in population S with relative incidences $p(a)=(p(a_1),\ldots,p(a_m))$, where $m\leq|S|$ is the number of distinct income levels in a, and where m is a natural number. The RD of an individual earning a_i is defined as the weighted sum of the excesses of incomes higher than a_i such that each excess is weighted by its relative incidence, namely

$$RD_s(a_i)=\sum_{a_j>a_i}p(a_j)(a_j-a_i). \tag{B1}$$

[6]The empirical findings support the relative income hypothesis. Duesenberry (1949) already found that individuals' levels of savings depend on their positions in the income distribution, and that the incomes of the richer people affect the behavior of the poorer ones (but not *vice versa*). Later on, and for example, Schor (1998) showed that, keeping annual and permanent income constant, individuals whose incomes are lower than the incomes of others in their community save significantly less than those in their community who are relatively better off.

In the example given above with income distribution {1,2,6,6,6,8}, we have that the vector of incomes is $a = (1, 2, 6, 8)$, and that the corresponding relative incidences are $p(a) = (1/6, 1/6, 3/6, 1/6)$. Therefore, the *RD* of the individual earning 2 is $\sum_{a_j > a_i} p(a_j)(a_j - a_i) = p(6)(6-2) + p(8)(8-2) = \frac{3}{6} \cdot 4 + \frac{1}{6} \cdot 6 = 3$. By similar calculations, we have that the *RD* of the individual earning 1 is higher at $3\frac{5}{6}$, and that the *RD* of each of the individuals earning 6 is lower at $\frac{1}{3}$.

We expand vector a to include incomes with their possible respective repetitions, that is, we include each a_i as many times as its incidence dictates, and we assume that the incomes are ordered, that is, $a = (a_i, \dots, a_{|S|})$ such that $a_i \le a_2 \le \dots \le a_{|S|}$. In this case, the relative incidence of each a_i, $p(a_i)$, is $1/|S|$, and (B1), defined for $i = 1, \dots, |S| - 1$, becomes

$$RD_s(a_i) \equiv \frac{1}{|S|} \sum_{j=i+1}^{|S|} (a_j - a_i). \tag{B2}$$

If we additionally assume that for $i = 1, \dots, |S|$, i denotes an individual whose income is a_i, then from (B2) we get that the *RD* of individual i is

$$RD_s(i) \equiv \frac{1}{|S|} \sum_{j=i+1}^{|S|} (a_j - a_i). \tag{B3}$$

For $i, j \,\epsilon \{1, \dots, |S|\}$ such that $j \le i$ it holds that $a_j - a_i \le 0$. Thus, for $i \,\epsilon \{1, \dots, |S|\}$, we obtain

$$\sum_{j=i+1}^{|S|} (a_j - a_i) = \sum_{j=i+1}^{|S|} (a_j - a_i) + i \cdot 0 = \sum_{j \in s} \max\{a_j - a_i, 0\}. \tag{B4}$$

Inserting (B4) into (B2) yields

$$RD_s(i) \equiv \frac{1}{|S|} \sum_{j \in s} \max\{a_j - a_i, 0\},$$

which aligns with Definition 2.